

## **The impact of range-dependent sediment properties on the acoustic field in 2-D shallow water environments**

Jason D. Sagers

Applied Research Laboratories at The University of Texas at Austin

Environmental Science Laboratory

Austin, TX 78758

phone: (512) 835-3195 fax: (512) 490-4225 email: [sagers@arlut.utexas.edu](mailto:sagers@arlut.utexas.edu)

Award Number: N00014-10-1-0650

### **LONG-TERM GOALS**

The goal of this research is to understand the effects of range-dependent sediment properties on the acoustic field in 2-D shallow water environments. This information, in part, is required to solve the statistical inference problem in inhomogeneous shallow water environments.

### **OBJECTIVES**

The objective of the current work is to create and benchmark a two-way coupled mode model that will later be used to analyze the effects of range-dependent sediment properties on the acoustic field in typical littoral environments. An accurate coupled mode model is required to properly capture the physics of shallow water acoustic propagation and to classify the propagation as belonging to either the adiabatic or coupled mode regime. It is of particular interest to classify acoustic data taken during the Shallow Water 2006 (SW06) experiment as being adiabatic or coupled mode and to identify the physical mechanisms that cause mode coupling.

### **APPROACH**

The model developed in this work is based on two-way coupled mode theory. The inhomogeneous Helmholtz equation for pressure  $p_\omega(z, \vec{r})$  due to a continuous wave (CW) point source located at  $(z_s, \vec{r}_s)$  is

$$\rho(z, \vec{r}) \nabla \cdot \left[ \frac{1}{\rho(z, \vec{r})} \nabla p_\omega(z, \vec{r}) \right] + k^2(z, \vec{r}) p_\omega(z, \vec{r}) = -4\pi p_0 \delta(\vec{r} - \vec{r}_s) \delta(z - z_s), \quad (1)$$

where  $k = 2\pi f/c(z, \vec{r})$  is the spatially-dependent acoustic wavenumber,  $z$  is the depth coordinate,  $\vec{r}$  is the horizontal coordinate,  $\rho$  is the spatially-dependent fluid density, and  $p_0$  is the CW source spectral amplitude referenced to 1 m. The fluid properties for  $c$  are allowed to be completely arbitrary in depth and range while the properties for  $\rho$  are allowed to be piecewise continuous in depth and arbitrary in range. Continuity of pressure at interfaces and continuity of particle velocity normal to fluid interfaces

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>2010</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2010 to 00-00-2010</b>	
4. TITLE AND SUBTITLE <b>The impact of range-dependent sediment properties on the acoustic field in 2-D shallow water environments</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>University of Texas at Austin, Applied Research Laboratory, Environmental Science Laboratory, Austin, TX, 78758</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			
			<b>Same as Report (SAR)</b>	<b>7</b>	

are properly enforced to conserve energy.<sup>1,2</sup> The solution for  $p_\omega(z, \vec{r})$  can be represented by the normal mode summation

$$p_\omega(z, \vec{r}) = \sum_{m=1}^M R_m(\vec{r}|z_s, \vec{r}_s) \phi_m(z, \vec{r}), \quad (2)$$

where  $M$  is the total number of modes included in the summation,  $R_m(\vec{r}|z_s, \vec{r}_s)$  are range-dependent modal amplitudes specific to the source position  $z_s$  and  $r_s$ , and  $\phi_m(z, \vec{r})$  are the depth dependent local mode functions. Note that in this work, the possibility of horizontal (out of plane) refraction is neglected. Therefore, the range coordinate  $\vec{r}$  becomes  $x$  or  $r$  for Cartesian or cylindrical geometries, respectively.

The major challenge of the two-way coupled mode model used in this research is to solve for the range-dependent modal amplitudes  $R_m(\vec{r}|z_s, \vec{r}_s)$  that appear in Eq. (2). This is done by transforming Eq. (1) into an integral equation of the form

$$R_m(\vec{r}|z_s, \vec{r}_s) = -4\pi p_0 \frac{\phi_m(z_s, \vec{r}_s)}{\rho(z_s)} G_m(\vec{r}, \vec{r}_s) - \int G_m(\vec{r}, \vec{r}') \left[ \sum_{n=1}^M (\mathbf{B}_{mn}(\vec{r}') \cdot \nabla_{\vec{r}'} + C_{mn}) R_n(\vec{r}'|z_s, \vec{r}_s) \right] d\vec{r}', \quad (3)$$

where  $G_m(\vec{r}, \vec{r}_s)$  is a two-way adiabatic Green's function and  $\mathbf{B}_{mn}(\vec{r})$  and  $C_{mn}$  are mode coupling matrices. The solution method for Eq. (3) is relatively efficient when using the Lanczos routine proposed by Knobles.<sup>3</sup> It should be noted that the coupled mode model developed here is capable of including both the trapped and continuum components of the modal spectrum.

## WORK COMPLETED

Much of the work completed was to implement the coupled mode model in MATLAB and then to benchmark the new model against other known models and accepted results. The new model was benchmarked for a number of cases, including problems with range-dependencies in the bathymetry, the water column, and the sediment geoacoustic properties. Through the benchmarking process, the model was shown to be in agreement with other model solutions, specifically a PE model and a two-way coupled mode model based on the differential equation form of the 2-D Helmholtz equation.

The integral equation coupled mode model was then applied to a typical propagation environment encountered during the SW06 experiment to see if coupled mode effects due to seabed inhomogeneities were present. The particular environment under analysis included variable bathymetry and sub-bottom interface depths. Range-independent sediment properties and water column sound speed profiles were assumed for the initial analysis, which is an assumption that may be suspect for the SW06 environment (consequently the initial analysis may underestimate the amount of mode coupling). However, even in this simplified environment, it was observed through the model calculation that coupled mode effects may be present in the measured acoustic data.

An initial attempt was made to match the modeled data with measured acoustic data on one of the vertical line arrays (VLA) deployed during SW06. A good initial fit between model and data was sought in order to make conclusive statements about whether coupled mode effects are present in the measured acoustic data. The effect of horizontal variability in the environment was shown to have a minimal effect on the differences between modeled and measured transmission loss.

## RESULTS

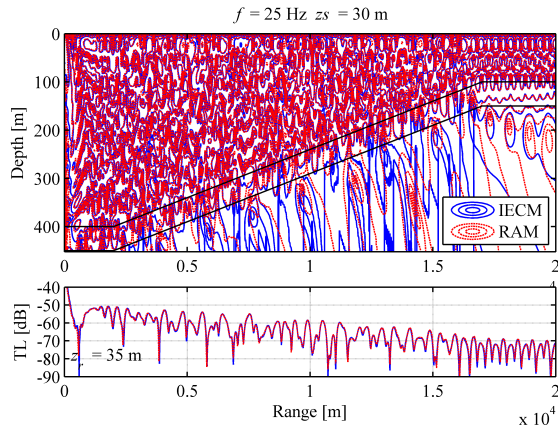
This section presents results for (A) the comparison of the coupled mode model to other propagation models for selected benchmark problems and (B) model-data comparisons for a propagation track in the SW06 experiment.

### A. Benchmark comparison

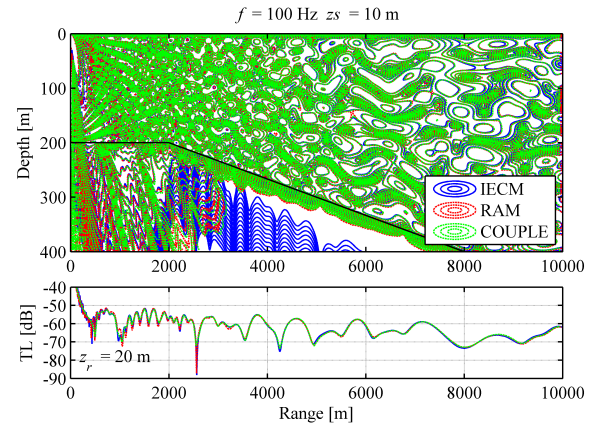
The results of two benchmark problems are shown here. Throughout this section the new integral equation coupled mode model will be designated by the acronym IECM, Collin's range-dependent acoustic model<sup>4</sup> will be designated by RAM, and Evan's coupled mode model<sup>5</sup> will be designated by COUPLE. Comparisons to RAM and COUPLE were made as these programs represent the current state of the art in range dependent propagation modeling.

1. The shelf break front problem was proposed as part of the SWAM99 workshop. The benchmarked case shown here is after the slightly modified problem published by Knobles.<sup>6</sup> Both the bathymetry and water column sound speed profile are range-dependent. The comparison between the IECM and RAM solutions are shown in Fig. 1, with contour lines drawn at 3 dB intervals throughout the range-depth plane. There is excellent agreement between the two models in the water column but discrepancies exist in the field inside the sediment layers. As is typical when using the Pekeris branch cut (which is done in this research when using ORCA to find the local mode depth functions), the unphysical behavior of the leaky modes in the sediment layers cause the field in the sediment to be in error.

2. Outing<sup>7</sup> proposed a benchmark problem involving a downslope. His Example A was slightly modified for the benchmark case shown here. A source is located at  $z_s = 10$  m in the water column emitting a CW sound at  $f = 100$  Hz. The comparison between the IECM, RAM, and COUPLE solutions are shown to be in excellent agreement in Fig. 2.



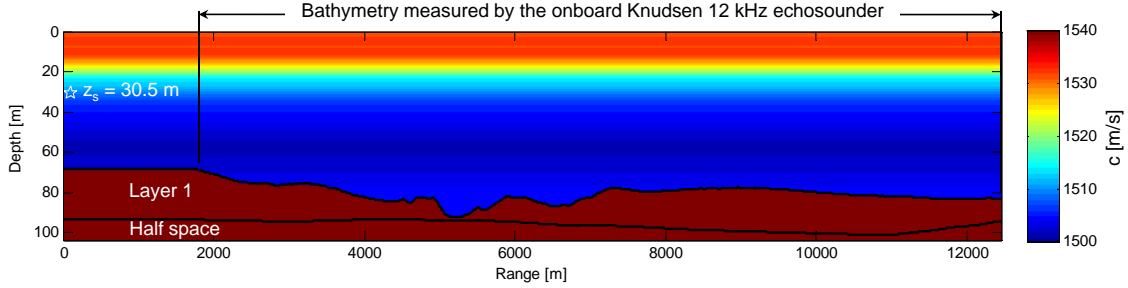
**Figure 1: Top: Contour plot of TL for the shelf break front problem. Bottom: TL at  $z_r = 35$  m.**



**Figure 2: Top: Contour plot of TL for Outing's Example A problem. Bottom: TL at  $z_r = 20$  m.**

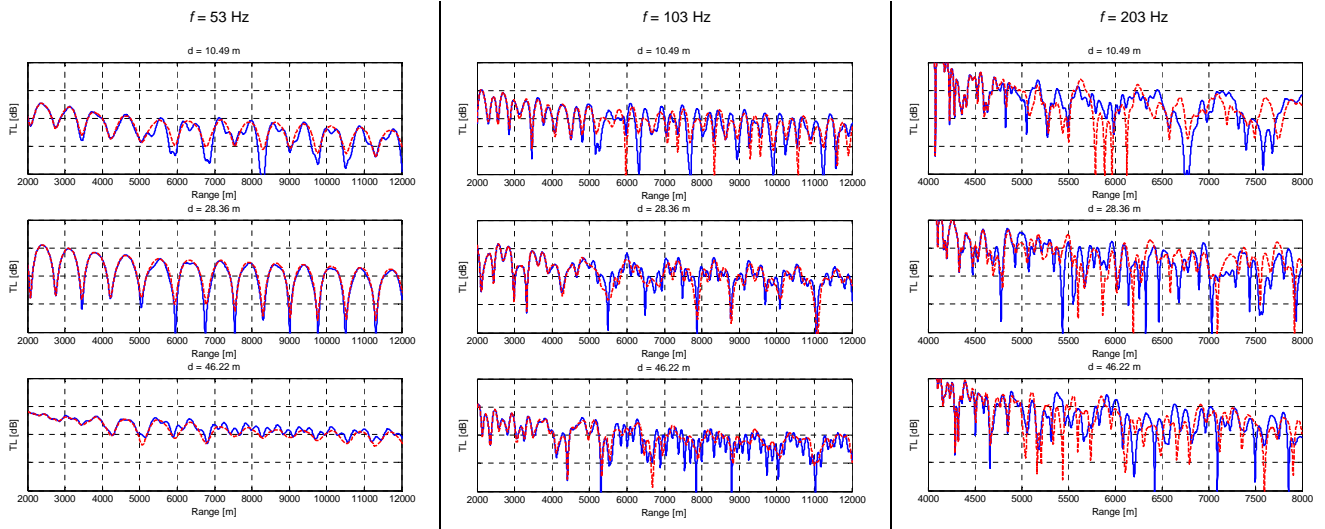
The IECM model was applied to an environment that occurred during the SW06 experiment as the R/V Knorr towed a J-15 source from 39.0457 N, 73.1362 W to 38.9923 N, 73.0172 W on August 26th between the hours of 02:15 to 03:30. The bathymetry underneath the Knorr was measured by an onboard Knudsen echosounder and is shown in Fig. 3. The variation in the bathymetry over the course of the track was approximately 20% of the mean water depth. The measured bathymetry was used to

create a representative acoustic propagation track between source and receiver. Nominal sound speed, density, and attenuation values of 1650 m/s, 1.6 g/cm<sup>3</sup>, and 0.08 dB/λ and 1725 m/s, 2.0 g/cm<sup>3</sup>, and 0.05 dB/λ were used for the first layer and halfspace, respectively. The average water column sound speed profile measured on the nearby Shark array over the time of the tow was used in the model.



**Figure 3: Bathymetry as measured by the onboard Knudsen echosounder for a down-shelf track from SWAMI 32.**

Transmission loss as a function of range was computed for three different frequencies (53, 103, and 203 Hz) and three different receiver depths (10.49, 28.36, and 46.22 m). Both the adiabatic and coupled mode solutions were computed with the IECM model. The comparisons of these two solutions are shown in Fig. 4. It is observed that the bathymetry variation present in this environment is sufficient to cause coupled mode propagation at these frequencies. It is also observed that the degree of mode coupling increases with frequency.

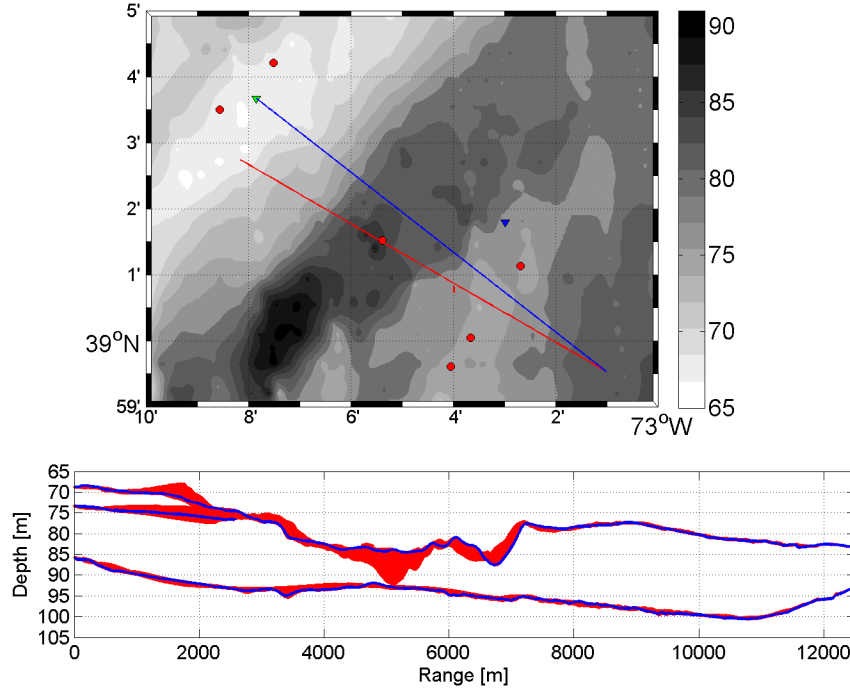


**Figure 4: Adiabatic (red) and IECM (blue) TL solutions for the SW06 environment at 53, 103, and 203 Hz and three different receiver depths.**

## B. Model-data comparison

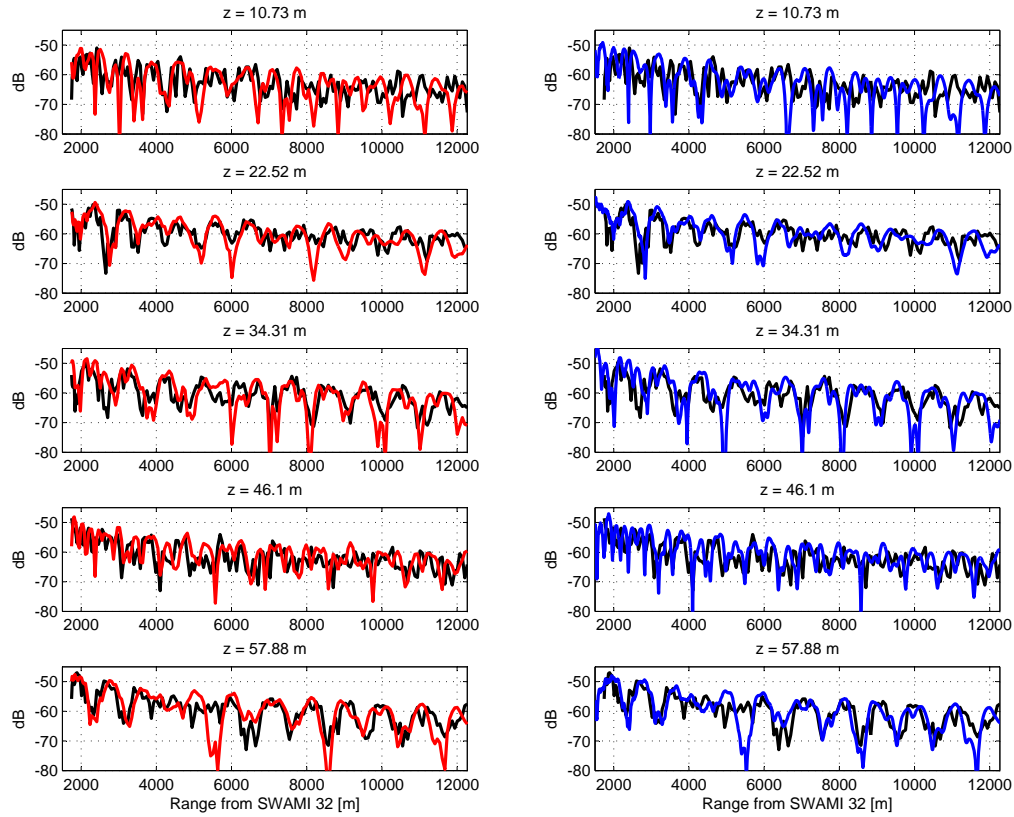
Through the efforts described above, it was discovered that coupled mode effects may be present, and even significant, in the SW06 environment. An effort was made to observe these effects in measured acoustic data. This first required a more careful construction of the acoustic propagation path between

source and receiver in order to decide whether misfit in the model-data transmission loss was caused by horizontal inhomogeneities. As is commonly done in propagation modeling, a single acoustic track was constructed between the receiver location (the SWAMI32 VLA located at 39.0611 N 73.1308 W) and the final position of the source along the tow track (38.9923 N, 73.0172 W). The common assumption is that horizontal variability in the environment is small enough to be neglected. This track will be called the “endpoint” track and is shown in blue in Fig. 5. The actual acoustic tracks propagate from each source position on the red track back to the SWAMI32 VLA. Each separate acoustic track is plotted in red on the bottom of Fig. 5 to illustrate horizontal bathymetry variations in this region.



**Figure 5: Top: Knorr ship track (red) and endpoint acoustic track (blue) plotted on top of bathymetry contours (grayscale). CTD measurements shown as dots. Bottom: Bathymetry and sub-bottom layer depths for multiple acoustic tracks (red) and endpoint acoustic track (blue).**

Because of its computational efficiency, the propagation model RAM was used to compute the transmission loss along 180 different acoustic propagation paths between the receiver and the moving source. RAM was also used to compute the transmission loss along the single endpoint track. The comparison between measured and modeled transmission loss at 103 Hz, while using nominal values for the sediment geoacoustic parameters, is shown in Fig. 6. The degree of misfit between model and data does not change significantly at 103 Hz by considering the horizontal variability in the environment. This is an important result for this particular track and suggests that the horizontal variability is small enough to neglect at and below this frequency. This will allow for a single-track computation with the IECM model (which is much more computationally intensive than RAM) to analyze the significance of the mode coupling. However, it is possible that the effect of horizontal variability will increase with frequency. This is an important consideration for future work, given that the degree of mode coupling was also observed to increase with frequency.



**Figure 6:** Comparison between measured data (black) and modeled data using 180 propagation paths (red) and the endpoint path (blue) at five receiver depths on the SWAMI32 VLA.

## IMPACT/APPLICATIONS

The impact of this research is to increase the understanding of physical propagation mechanisms in shallow water environments and to identify important modeling considerations that must be accounted for to decrease uncertainty in model prediction. The research can be applied to propagation models of all types and can also be used in geoacoustic inversion and inference problems.

## TRANSITIONS

The primary transition for this project is an accurate 2-D coupled mode model for acoustic propagation in shallow water environments, and a methodology for statistical inference for seabed parameters in environments that have horizontal and range variability.

## RELATED PROJECTS

None.

## REFERENCES

- [1] S. R. Rutherford and K. E. Hawker, “Consistent coupled mode theory of sound propagation for a class of nonseparable problems”, *The Journal of the Acoustical Society of America* **70**, 554–564 (1981).
- [2] J. A. Fawcett, “A derivation of the differential equations of coupled-mode propagation”, *The Journal of the Acoustical Society of America* **92**, 290–295 (1992).
- [3] D. P. Knobles, S. A. Stotts, R. A. Koch, and T. Udagawa, “Integral equation coupled mode approach applied to internal wave problems”, *Journal of Computational Acoustics* **9**, 149 (2001).
- [4] M. D. Collins and E. K. Westwood, “A higher-order energy-conserving parabolic equation for range-dependent ocean depth, sound speed, and density”, *The Journal of the Acoustical Society of America* **89**, 1068–1075 (1991).
- [5] R. B. Evans, “A coupled mode solution for acoustic propagation in a waveguide with stepwise depth variations of a penetrable bottom”, *The Journal of the Acoustical Society of America* **74**, 188–195 (1983).
- [6] D. P. Knobles, S. A. Stotts, and R. A. Koch, “Low frequency coupled mode sound propagation over a continental shelf”, *The Journal of the Acoustical Society of America* **113**, 781–787 (2003).
- [7] D. A. Outing, W. L. Siegmann, M. D. Collins, and E. K. Westwood, “Generalization of the rotated parabolic equation to variable slopes”, *The Journal of the Acoustical Society of America* **120**, 3534–3538 (2006).